

Superluminal and Ultraslow Light Propagation in an Optomechanical Systems

Devrim Tarhan¹, Sumei Huang², and Özgür E. Müstecaplıoğlu³

¹ *Department of Physics, Harran University, Osmanbey Yerleşkesi, Şanlıurfa, 63300, Turkey*

² *Department of Physics, University of California, Merced, California 95343, USA and*

³ *Department of Physics, Koç University, Sarıyer, Istanbul, 34450, Turkey**

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We consider an optomechanical double-ended cavity under the action of a coupling laser and a probe laser in electromagnetically induced transparency configuration. It is shown how the group delay and advance of the probe field can be controlled by the power of the coupling field. In contrast to single-ended cavities, only allowing for superluminal propagation, possibility of both superluminal and subluminal propagation regimes are found. The magnitudes of the group delay and the advance are calculated to be ~ 1 ms and ~ -2 s, respectively, at a very low pumping power of a few microwatts. In addition, interaction of the optomechanical cavity with a time dependent probe field is investigated for controlled excitations of mirror vibrations.

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I. INTRODUCTION

Demonstration of ultraslow group velocity (v_g) of light [1] in ultracold atoms by electromagnetically induced transparency (EIT) [2] has inspired appealing applications [3–8]. Besides the slow light, superluminal phenomena ($v_g > c$ or v_g is negative) was observed in atomic caesium gas [9, 10] and in alexandrite crystal [11]. Slow and superluminal light have also been observed in optomechanical systems [12] whose superior delay and advancement times, smaller dimensions, and less demanding thermal requirements makes them attractive for quantum optomechanical memory and classical signal processing applications [13–22]. Recent proposal such as optomechanical cavity with a Bose-Einstein condensate (BEC) [23] and one-sided cavity with a nanomechanical mirror (NMM) [24], which is recently demonstrated [21], are promising but either too costly and difficult to implement [23] or not sufficiently flexible enough to realize both superluminal and slow light effects simultaneously [21, 24]. The analogue of electromagnetically induced transparency has been demonstrated very recently in a room temperature cavity optomechanics setup formed by a thin semitransparent membrane within a Fabry-Perot cavity [25]. We address the question of how more controllable and simpler optomechanical systems, that can simultaneously exhibit larger delay and advancement times, can be realized.

In this work, we investigate the time delay of the weak probe field at the probe resonance in a high quality cavity NMM under the action of coupling laser. We find that the group delay can be controlled by the power of the coupling field. The time delay is positive which corresponds to ultraslow light propagation (subluminal propagation) when there is a strong coupling between the nano-oscillator and the cavity. In contrast to single-

ended cavities, only allowing for superluminal propagation [26], possibility of both superluminal and subluminal propagation regimes are found. The magnitude of the group delay is ~ 2 ms at a very low pumping power of a few microwatts. The transmission group delay that we have found is larger than the group delay in a coupled BEC-cavity system [23] which is costly and difficult system to implement. In addition, we show that it is possible to control the vibrational excitations of the NMM by time dependent probe field.

Organization of the paper is as follows. In Sec. II we describe our physical optomechanical system and EIT configuration. The quantities such as group delay and advancement times redefined here as well. Results are given in Sec. III in two sub-sections. The first sub-section is dedicated to the case of constant pump and probe fields while the other one focuses on the case of time-dependent fields. Conclusion is given in Sec. IV.

II. MODEL SYSTEM

We consider the classical probe field ε_p and calculate the response of the cavity optomechanical system to the probe field in the presence of the coupling field ε_c . The nanomechanical oscillator of frequency ω_m is coupled to a Fabry-Perot cavity via radiation pressure effects [14]. In a Fabry-Perot cavity, both mirrors have equal reflectivity. We use a configuration in which a partially transparent NMM is in the middle of a cavity that is bounded by two high-quality mirrors as shown in Fig. 1. The system is driven by a coupling field of frequency ω_c and the probe field has frequency ω_p . The Hamiltonian of this system is given by

$$H = \hbar(\omega_0 - \omega_c)c^\dagger c + \hbar g c^\dagger c q + \frac{p^2}{2m} + \frac{1}{2}m\omega_m^2 q^2 + i\hbar\varepsilon_c(c^\dagger - c) + i\hbar(c^\dagger \varepsilon_p e^{-i\delta t} - c \varepsilon_p^* e^{i\delta t}), \quad (1)$$

where $\delta = \omega_p - \omega_c$, $g = -\omega_c/L$ is the coupling constant between the cavity field and the movable mirror [30], and

*Electronic address: dtarhan@harran.edu.tr

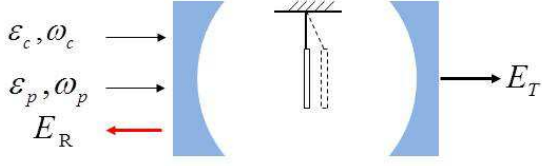


FIG. 1: Schematic of a double-ended cavity with a moving nanomechanical mirror adapted from Ref. [14].

c, c^\dagger are the annihilation creation and operators of the photons of the cavity field respectively. The momentum and position operators of the nanomechanical oscillator are p and q , respectively. The amplitude of the pump field is $|\varepsilon_c| = \sqrt{2\kappa P_c / \hbar \omega_c}$ with P_c being the pump power.

Heisenberg equation of motion for the coupled cavity-mirror system is written with phenomenological adding the damping rate 2κ representing loss at the cavity mirrors. The system is examined in the mean field limit [24]

$$\begin{aligned} \langle \dot{q} \rangle &= \frac{\langle p \rangle}{m}, \\ \langle \dot{p} \rangle &= -m\omega_m^2 \langle q \rangle - \hbar g \langle c^\dagger \rangle \langle c \rangle - \gamma_m \langle p \rangle, \\ \langle \dot{c} \rangle &= -[2\kappa + i(\omega_0 - \omega_c + g \langle q \rangle)] \langle c \rangle + \varepsilon_c + \varepsilon_p e^{-i\delta t}, \\ \langle \dot{c}^\dagger \rangle &= -[2\kappa - i(\omega_0 - \omega_c + g \langle q \rangle)] \langle c^\dagger \rangle + \varepsilon_c^* + \varepsilon_p^* e^{i\delta t}. \end{aligned} \quad (2)$$

The linear response solution is developed analytically using the ansatz [31],

$$\begin{aligned} q(t) &= q_0 + q_+ e^{-i\delta t} + q_- e^{i\delta t}, \\ p(t) &= p_0 + p_+ e^{-i\delta t} + p_- e^{i\delta t}, \\ c(t) &= c_0 + c_+ e^{-i\delta t} + c_- e^{i\delta t}, \end{aligned} \quad (3)$$

where q_0, p_0 and c_0 are the zeroth order solutions while the next terms corresponds to the first order solutions in probe field amplitude. By inserting Eq. 3 into the Heisenberg equation of motion we first obtain the steady state solutions $c_0 = \varepsilon_c / (2\kappa + i\Delta)$ and $q_0 = -\hbar g |c_0|^2 / m\omega_m^2$. Using them the first order solutions are analytically determined to be

$$c_+(\delta) = \frac{m(\delta^2 - \omega_m^2 + i\gamma_m\delta)[2\kappa - i(\Delta + \delta)] - i\alpha}{m(\delta^2 - \omega_m^2 + i\gamma_m\delta)[(2\kappa - i\delta)^2 + \Delta^2] + 2\Delta\alpha}, \quad (4)$$

where $\Delta = \omega_0 - \omega_c + gq_0$ is the effective detuning and $\alpha = \hbar g^2 |c_0|^2$. $|c_0|^2$ is the resonator intensity and q_0 is the steady state position of the movable mirror. $\varepsilon_T = 2\kappa c_+(\delta)$ is the transmitted and $\varepsilon_R = 2\kappa c_+(\delta) - 1$ is the reflected components of the probe field.

We can write the output field $\varepsilon_{out}(t) = \varepsilon_{out0} + \varepsilon_{out+} e^{-i\delta t} + \varepsilon_{out-} e^{i\delta t}$ [31]. Inserting this to the input-output relation and comparing the first order according to the ε_p , we get the probe response $(\varepsilon_{out+} + 1) = 2\kappa c_+$. The reflection and the transmission of the probe response are denoted by ε_R and ε_T , respectively. The reflection and transmission of the output field respectively are determined by $E_R = \varepsilon_R \varepsilon_p e^{-i\omega_p t}$. and

$E_T = \varepsilon_T \varepsilon_p e^{i\omega_p t}$. The amplitude of the transmission output field is $E_T = |T| \exp(i\phi(\omega_p))$.

If we expand $\phi(\omega_p)$ around $\bar{\omega}$ to the first order

$$\phi(\omega_p) = \phi(\bar{\omega}) + (\omega_p - \bar{\omega}) \frac{\partial \phi}{\partial \omega_p} \Big|_{\bar{\omega}}, \quad (5)$$

the transmitted probe pulse can be expressed as $|T| \varepsilon_p e^{-i\omega_p t} e^{i\phi(\bar{\omega})} e^{i(\omega_p - \bar{\omega}) \frac{\partial \phi}{\partial \omega_p} \Big|_{\bar{\omega}}}$, where $\phi(\bar{\omega}) = 0$ at resonance. Combining with the $e^{-i\omega_p(t-\tau)}$, the transmitted probe pulse peaks at $t = \tau$, where τ is the pulse delay that can be defined as

$$\tau = \left[\frac{\partial \phi}{\partial \omega_p} \right] \Big|_{\bar{\omega}}. \quad (6)$$

Phase of the output field can be found as

$$\phi = \frac{1}{2i} \ln \left(\frac{\varepsilon_T}{\varepsilon_T^*} \right). \quad (7)$$

The time delay of the transmission and reflection pulse can be determined by

$$\begin{aligned} \tau_T &= \text{Im} \left[\frac{1}{\varepsilon_T} \frac{\partial \varepsilon_T}{\partial \omega_p} \right] \Big|_{\bar{\omega}} \\ \tau_R &= \text{Im} \left[\frac{1}{\varepsilon_R} \frac{\partial \varepsilon_R}{\partial \omega_p} \right] \Big|_{\bar{\omega}} \end{aligned} \quad (8)$$

III. RESULTS AND DISCUSSIONS

In our calculations, we use parameters [19] for the length of the cavity $L = 6.7$ cm, the wavelength of the laser $\lambda = 2\pi c / \omega_c = 1064$ nm, $m = 40$ ng, $\omega_m = 2\pi \times 134$ kHz, $\gamma = 0.76$ Hz, $\kappa = \omega_m / 10$ and mechanical quality factor $Q = 1.1 \times 10^6$, and $\Delta = \omega_m$. The real and the imaginary parts of the $(\varepsilon_T = 2\kappa c_+)$ represent the absorptive and dispersive behavior, respectively.

A. Constant Pump and Probe Fields

We show the real and the imaginary parts of the ε_T In Fig. 2. Under the conditions of electromagnetically induced transparency in the mechanical system contained in a high quality cavity the system gives rise to dispersion that leads to ultraslow propagation of the probe field [14]. The phase is determined by Eq. 7 and ε_T , and the result is plotted in Fig. 4 as a function of the scaled dimensionless frequency δ / ω_m for the input coupling laser power $P_c = 1 \mu\text{W}$.

In the case of no coupling field $g = 0$, the delay time becomes $\tau_0 = 1.48 \mu\text{s}$. The coupling reverses the behavior of the system and the group delay becomes positive. We plot the group delay τ as a function of the pump power in Fig. 4 and Fig. 5 which show the group delay τ as a function of the pump power P_c . The group delay decreases with increasing power of the coupling field. The

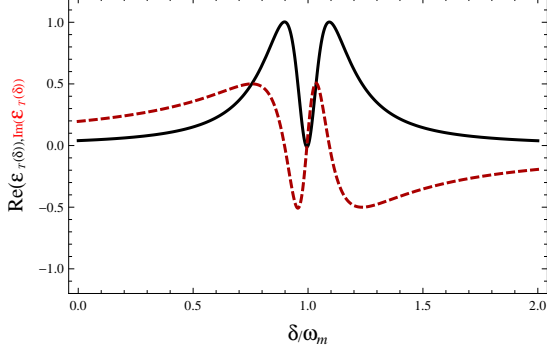


FIG. 2: (Color online) The the real (black solid) and the imaginary (red dashed) parts of the $\varepsilon_T(\delta)$ as a function of δ for input coupling laser power of $P_c = 5\mu\text{ W}$. The parameters used are the length of the cavity $L = 6.7\text{ cm}$, the wavelength of the laser $\lambda = 2\pi c/\omega_c = 1064\text{ nm}$, $m = 40\text{ ng}$, $\omega_m = 2\pi \times 134\text{ kHz}$, $\gamma = 0.76\text{ Hz}$, $\kappa = \omega_m/10$ and mechanical quality factor $Q = 1.1 \times 10^6$, and $\Delta = \omega_m$.

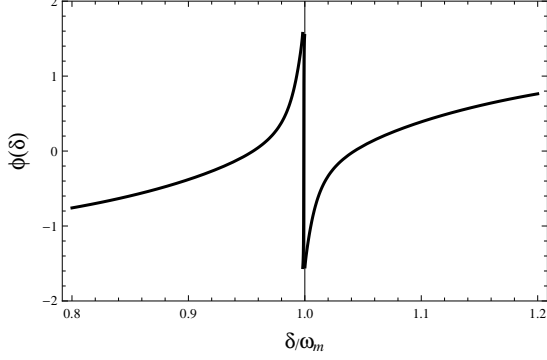


FIG. 3: Phase as a function of frequency δ for input coupling laser power $P_c = 1\mu\text{ W}$. The parameters are the same as Fig. 2.

probe pulse delay can be tuned by calibrating the pump power in the probe resonance ($\delta = \omega_m$) and $\Delta = \omega_m$. The pump power that we have used in Fig. 4 and Fig. 5 is on the order of $0.1 - 5\mu\text{W}$. We find large positive group delays of order 2 ms in a Fabry-Perot cavity under the action of a coupling laser and a probe laser. In Fig. 4 and Fig. 5 the group delays are positive, as a result the slow light effect can be observed. This corresponds to a subluminal situation. The physics of subluminal or superluminal light propagation in double-ended cavity optomechanical system is associated with the interaction of NNM and cavity field.

We plot the reflection $R(\delta) = |2\kappa c_+ - 1|^2$ and transmission spectrums $T(\delta) = |2\kappa c_+|^2$ of the probe field respectively. in Fig. 6 and Fig. 7. The width of the transparency window of EIT is given by [14]:

$$\Gamma(P_c) = \frac{\gamma_m}{2} + \frac{\alpha(P_c)}{4m\omega_m\kappa}, \quad (9)$$

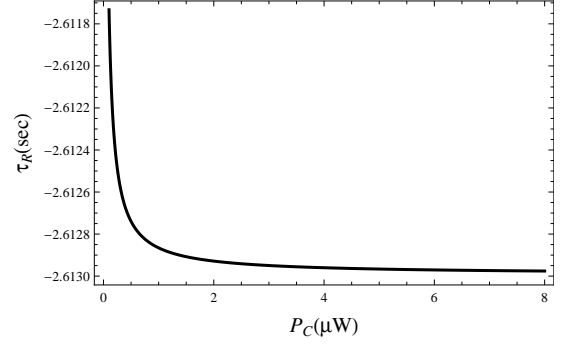


FIG. 4: Group delay as a function of the pump power. Time delay of the probe in the presence of the coupling field as a function of the power of the pumping or coupling field with $\Delta = \omega_m$. All parameters are the same with those of Fig. 2.

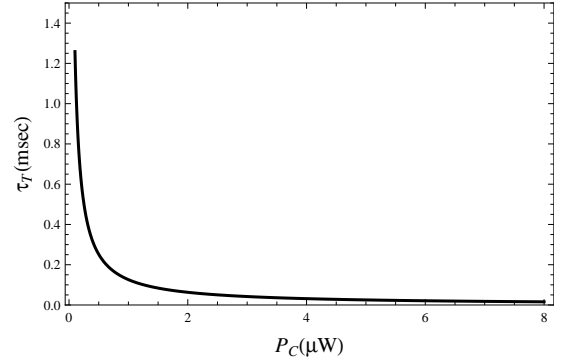


FIG. 5: Group delay as a function of the pump power. Time delay of the probe in the presence of the coupling field as a function of the power of the pumping or coupling field with $\Delta = \omega_m$. All parameters are the same with those of Fig. 2.

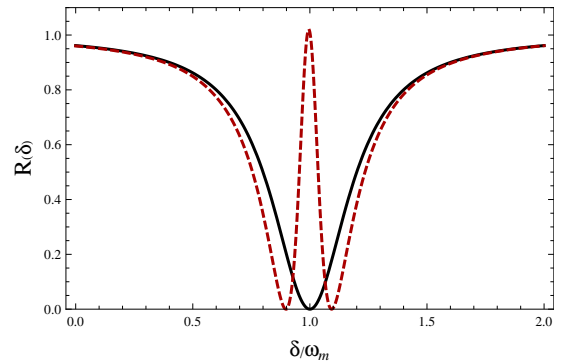


FIG. 6: (Color online) The reflection spectrum $R(\delta)$ as a function of normalized frequency with $\Delta = \omega_m$. $P_c = 0$ solid, $5\mu\text{W}$ (dashed). All parameters are the same with those of Fig. 2.

where $\alpha(P_c) = \hbar g^2 |c_0|^2$. EIT width changes with the

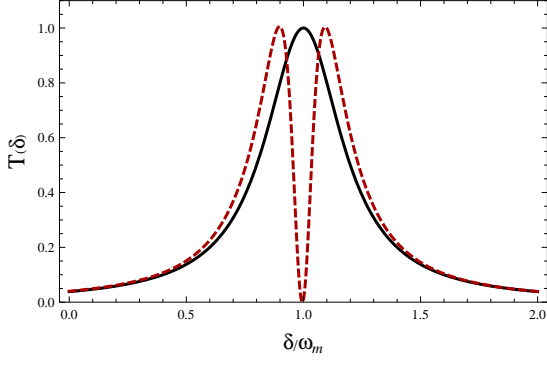


FIG. 7: (Color online) The transmission spectrum $R(\delta)$ as a function of normalized frequency with $\Delta = \omega_m$. $P_c = 0$ solid, $5\mu\text{W}$ (dashed). All parameters are the same with those of Fig. 2.

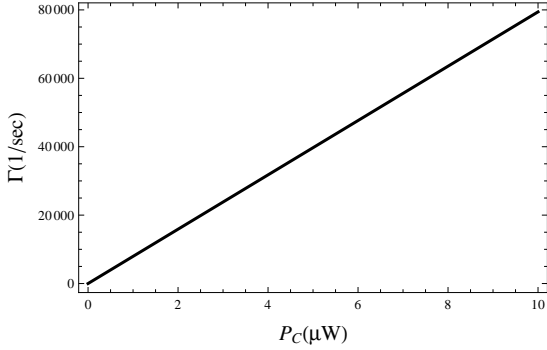


FIG. 8: EIT width $\Gamma(\delta)$ as a function of power $P_c = 0$ with $\Delta = \omega_m$. All parameters are the same with those of Fig. 2.

power in linear manner as shown in Fig. 8.

B. Time Dependent Probe Field

We now consider the time dependent, pulsed, probe field. As the dynamics of the optomechanical system is associated with the normal modes of the cavity field and the mirror vibrations, it is natural to expect the oscillation modes of the mirror can be controlled with temporal profile of the optical fields. We examine particularly pulses with duration much less than characteristic time of mirror oscillations. It can be interpreted as if the mirror oscillator is kicked by the optical pulses in sudden perturbations. We find the situations of both robust excitations of mirror motional modes where the mirror is simply displaced without oscillations and the periodical excitations where the mirror vibrates.

Now we use the following ansatz in terms of time dependence in order to obtain resonator-mirror coupled

equations:

$$\begin{aligned} q(t) &= q_0(t) + q_+(t)e^{-i\delta t} + q_-(t)e^{i\delta t}, \\ p(t) &= p_0(t) + p_+(t)e^{-i\delta t} + p_-(t)e^{i\delta t}, \\ c(t) &= c_0(t) + c_+(t)e^{-i\delta t} + c_-(t)e^{i\delta t}. \end{aligned} \quad (10)$$

Assuming $\dot{p}_+(t) = 0$ and $\dot{c}_-(t) = 0$, substituting Eq. 10 into Eq. 2, we obtain

$$\begin{aligned} \frac{dq_+}{dt} &= \frac{1}{s} \{d - i\hbar g^2 |c_0|^2\} q_+ - \left[\frac{\hbar g}{m(\gamma_m - i\delta)} c_0^* \right] c_+ \\ \frac{dc_+}{dt} &= -[2\kappa + i(\Delta - \delta)] c_+ - igc_0 q_+ + \varepsilon_p(t), \end{aligned} \quad (11)$$

where $s = m(\gamma_m - i\delta)[2\kappa - i(\Delta + \delta)]$, $d = m[2\kappa - i(\Delta + \delta)](i\delta\gamma_m - \delta^2 - \omega_m^2)$, and $\Delta = \omega_0 - \omega_c + gq_0$ is the effective detuning. The zeroth order solutions are $c_0 = \varepsilon_c/(2\kappa + i\Delta)$ and $q_0 = -\hbar g |c_0|^2 / m\omega_m^2$. Eq. 11 describes the coupled, normal mode excitations of mirror and optical modes propagation of probe field in a nanomechanical system. One can solve Eq. 11 by introducing the matrix notation

$$\vec{V} = \begin{pmatrix} q_+ \\ c_+ \end{pmatrix}, \quad (12)$$

$$\vec{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad (13)$$

$$\vec{F} = \begin{pmatrix} 0 \\ \varepsilon_p(t) \end{pmatrix}, \quad (14)$$

where $A = (-d + i\hbar g^2 |c_0|^2)/s$, $B = \hbar g c_0^* / m(\gamma_m - i\delta)$, $C = igc_0$, and $D = 2\kappa + i(\Delta - \delta)$. Eq. 11 becomes

$$\frac{d}{dt} \vec{V} = -\vec{M} \cdot \vec{V} + \vec{F}(t), \quad (15)$$

whose solution can be expressed as [32]:

$$\vec{V}(t) = e^{-\vec{M}(t-t_0)} \vec{V}(t_0) + \int_{t_0}^t e^{-\vec{M}(t-t')} \vec{F}(t') dt'. \quad (16)$$

If we take $t_0 \rightarrow -\infty$ the solution becomes :

$$\vec{V}(t) = \int_{-\infty}^t e^{-\vec{M}(t-t')} \vec{F}(t') dt'. \quad (17)$$

If $\vec{F}(t')$ is constant the steady state solution $\vec{V} = \vec{M}^{-1} \vec{F}$ and $\dot{\vec{V}} = 0$. We take the pump field constant, whereas the probe field depends on time. After solving Eq. 11 analytically, we find $c_+(t)$ and $q_+(t)$ in terms of hypergeometric functions and the final result is plotted in Fig. 9 under the EIT condition of $\Delta = \omega_m$. We plot mirror vibrations($q_+(t)$) as a function of time in Fig. 9 for $\varepsilon_p(t) = \text{sech}(t)$. The total displacement of the robust excitations of NMM is:

$$q(t) = q_0 + 2q_+(t) \cos \delta t. \quad (18)$$

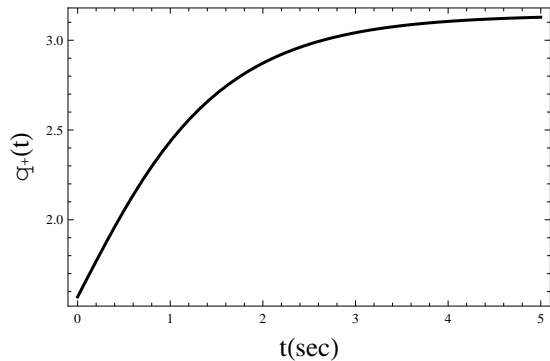


FIG. 9: $q_+(t)$ as a function of time with $\Delta = \omega_m$. All parameters are the same with those of Fig. 2.

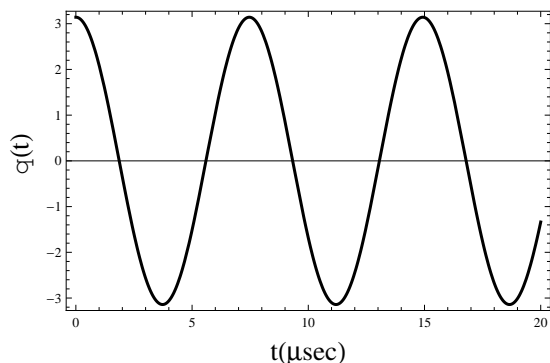


FIG. 10: Time dependence of $q(t)$ for $\Delta = \omega_m$ and $\delta = \omega_m$. All parameters are the same with those of Fig. 2.

We find the situations of both robust excitations of NMM motional modes where the mirror is simply displaced thanks to optical intensity. This behavior is dis-

played in Fig. 10. During the final stages of this work, similar time dependent control procedures are considered for optomechanical quantum memory applications [15].

IV. CONCLUSION

We have examined the question of delay and advance of the probe field under the conditions of electromagnetically induced transparency in optomechanical system contained in a high quality double-ended cavity. We have shown that it is possible to control the propagation of probe pulse in a double-ended cavity with a NNM. We have computed the transmission and reflection spectrum of the probe field. Tunable group delay and advance of optical pulse by adjusting the pump power are found. As the pump power increases the group delay becomes smaller, while it saturates beyond a critical value of the pump power. The magnitude of the group delay is found to be ~ 1 ms and the advance is ~ -2 s at a low pump power $\sim 0.2 \mu\text{W}$ for the parameters chosen as in Ref. [18]. The system under consideration is easier to implement and offers longer group delays in comparison to other optomechanical proposals [23]. Moreover, we have investigated the interaction of the optomechanical cavity with a time dependent probe field for controlled excitations of mirror vibrations and therefore, we have showed that thanks to optical intensity, mechanical mode excitations can be done due to time dependent probe field.

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